Introduction

Stability margin is an important index of control system. It represents the stability of the system. But it does not reflect the impact of various uncertainties on the system. Therefore, it is necessary to re-recognize the stability margin in order to put forward a better definition and discover new concepts.

1. Definition of Classic Stability Margin

Consider the unit feedback system shown in Figure 1.

Assuming that the open-loop transfer function of the system is G, Introducing Measurement Link \( L = ke^{j\theta} \), Measure the stability margin of the system.

Assuming that the above system is stable when \( L = 1 \), gain margin and phase margin are defined as follows:

**Gain margin \( K_{\text{min}} \) & \( K_{\text{max}} \):** If the closed-loop system is stable for all \( L = k \), \( K_{\text{min}} < k < K_{\text{max}} \), But \( L = K_{\text{min}} \) and \( L = K_{\text{max}} \) are unstable. Among \( 0 \leq K_{\text{min}} \leq 1, K_{\text{max}} \geq 1 \).

**Phase margin \( \theta_{\text{min}} \) & \( \theta_{\text{max}} \):** If the closed-loop system is stable for all \( L = e^{j\theta} \), \( \theta_{\text{min}} < \theta < \theta_{\text{max}} \). But \( L = ke^{j\theta_{\text{min}}} \) and \( L = ke^{j\theta_{\text{max}}} \) are unstable. Among \( -\pi \leq \theta_{\text{min}} \leq 0, 0 \leq \theta_{\text{max}} \leq \pi \).

Because the above definition of stability margin has clear physical meaning and simple calculation, it has been widely accepted and applied.

2. For SISO Systems

Consider a simple dynamic system:

**Example 1:** \( G(s) = \frac{s - 2}{2s - 1} \)

Four stabilizing controller:
Lack of Classic Stability Margin

\[ K_1(s) = 1 \]
\[ K_2(s) = \frac{s + 1.9}{1.9s + 1} \]
\[ K_3(s) = \frac{1.9s + 1}{s + 1.9} \]
\[ K_4(s) = \frac{s + 2.5}{2.5s + 1} \frac{1.7s^2 + 1.5s + 1}{s^2 + 1.5s + 1} \]

Figures 2 and 3 show Nyquist and Bode diagrams of the four systems. As can be seen from the figure:

For the stabilizer \( K_1 \), the system has larger gain margin and phase margin. The open-loop amplitude-phase curve is far from the critical point (-1, j0), which shows that the system has good robustness.

For the stabilizer \( K_2 \), the system has a large gain margin and a very small phase margin.

For the stabilizer \( K_3 \), the system has a very small gain margin and a large phase margin, but its open-loop amplitude-phase curve is very close to the critical point (-1, j0). It shows that the robustness of the system is poor. It is explained that the only gain margin or only phase margin is not a sufficient indicator of the robustness of the system.

For the stabilizer \( K_4 \), the gain margin and phase margin of the system are very large. But its open-loop amplitude-phase curve is very close to the critical point (-1, j0). It shows that the robustness of the system is poor. It shows that sometimes the combination of gain margin and phase margin may not be enough to indicate the real robustness of the system.

The problem is that the gain margin and phase margin can not correctly reflect the shortest distance from the critical point (-1, j0) of the open-loop amplitude-phase curve.

\textbf{Figure 2. Nyquist diagrams of four systems}
2. For MIMO Systems

For MIMO systems, the robustness of the system cannot be guaranteed even if its characteristic trajectory (equivalent to Nyquist diagram or open-loop amplitude-phase curve of SISO system) is far from the critical point (-1, j0). The following example illustrates this well.

Example 2: The transfer function matrix for the MIMO control system is:

$$G(s) = \frac{1}{(s + 1)(s + 2)} \begin{bmatrix} -47s + 2 & -56s \\ 42s & 50s + 2 \end{bmatrix}$$

By introducing $V = \begin{bmatrix} 7 & 8 \\ 6 & 7 \end{bmatrix}$, $V^{-1} = \begin{bmatrix} 7 & -6 \\ -6 & 7 \end{bmatrix}$, make $G(s)$ diagonally dominant

$$G_1 = VGV^{-1} = \begin{bmatrix} \frac{1}{s + 1} & 0 \\ 0 & \frac{2}{s + 2} \end{bmatrix}$$

$G_1$ is completely decoupled. Its characteristic function is:

$$g_1(s) = \frac{1}{s + 1}$$

$$g_2(s) = \frac{2}{s + 2}$$

The system’s characteristic trajectory is the circle on the right half-complex plane (see Figure 4). It is impossible to touch the left half plane, let alone close to the (-1, j0) point. The control system corresponding to each diagonal element has infinite gain margin and 90 degree phase margin. From the point of view of SISO stability margin,

Figure 3. Bode diagrams of four systems
Lack of Classic Stability Margin

the system should be very stable. However, if the first column of $G(s)$ is multiplied by 1.07 (gain increase 7%) and the second column is multiplied by 0.93 (gain decrease 7%), the system becomes closed-loop unstable.

This shows that the Nyquist theorem of multivariable systems is not suitable for judging the robustness of multivariable systems. Analysis of the MIMO system using the SISO method may lead to incorrect conclusions.

![Nyquist Diagram](image)

**Figure 4. Nyquist diagram of example 2 system**

**CONCLUSION**

As can be seen from the above, the classical stability margin has the following lack:

1) For SISO systems, it does not reflect the minimum distance from the system Nyquist curve to the critical point (-1, j0). Or it can not reflect the influence of uncertainties that gain and phase change simultaneously on the system.

2) For MIMO systems, Even if each loop has a good stability margin, when both loops have uncertainties at the same time, the system can easily become unstable. It can not reflect the influence of multi-loop uncertainty on MIMO system.

In order to overcome these lacks, new concepts and indicators are needed to characterize the characteristics of the system.

**REFERENCES**


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