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Lack of Classic Stability Margin

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Abstract: The gain margin and phase margin defined by classical control theory are for SISO systems. It is not enough for the robustness of the system. If it is used in the MIMO system, an incorrect conclusion may be given. This paper illustrates the above conclusions with examples.

Keywords: Stability margin, Robustness, Uncertainty.

Introduction

Stability margin is an important index of control system. It represents the stability of the system. But it does not reflect the impact of various uncertainties on the system. Therefore, it is necessary to re-recognize the stability margin in order to put forward a better definition and discover new concepts.

1. Definition of Classic Stability Margin

Consider the unit feedback system shown in Figure 1.

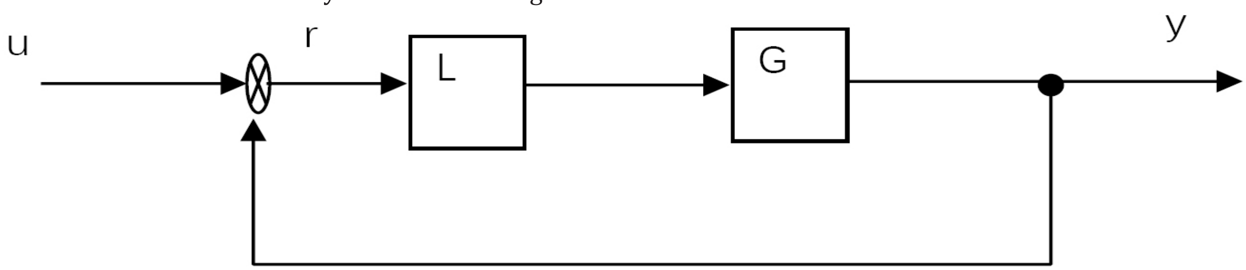

Figure1. *Block diagram of unit feedback control system*

Assuming that the open-loop transfer function of the system is G, Introducing Measurement Link $L = ke^{j\theta}$, Measure the stability margin of the system.

Assuming that the above system is stable when L = 1, gain margin and phase margin are defined as follows:

Gain margin K_{min} & K_{max}: If the closed-loop system is stable for all L=k, Kmin<k<Kmax, But L = Kmin and L = Kmax are unstable. Among $0 \le K_{min} \le 1, K_{max} \ge 1$.

Phase margin θ_{min} & θ_{max}: If the closed-loop system is stable for all $L = e^{j\theta}$, $\theta_{min} < \theta < \theta_{max}$, But $L = ke^{j\theta min}$ and $L = ke^{j\theta max}$ are unstable. Among $-\pi \le \theta_{min} \le 0$, $0 \le \theta_{max} \le \pi$.

Because the above definition of stability margin has clear physical meaning and simple calculation, it has been widely accepted and applied.

2. For SISO Systems

Consider a simple dynamic system:

Example 1: $G(s) = \dfrac{s-2}{2s-1}$

Four stabilizing controller:

$$K_1(s) = 1$$

$$K_2(s) = \frac{s+1.9}{1.9s+1}$$

$$K_3(s) = \frac{1.9s+1}{s+1.9}$$

$$K_4(s) = \frac{s+2.5}{2.5s+1}\frac{1.7s^2+1.5s+1}{s^2+1.5s+1.7}$$

Figures 2 and 3 show Nyquist and Bode diagrams of the four systems. As can be seen from the figure:

For the stabilizer K_1, the system has larger gain margin and phase margin. The open-loop amplitude-phase curve is far from the critical point (-1, j0), which shows that the system has good robustness.

For the stabilizer K_2,The system has a large gain margin and a very small phase margin.

For the stabilizer K_3, The system has a very small gain margin and a large phase margin, But its open-loop amplitude-phase curve is very close to the critical point (-1, j0). It shows that the robustness of the system is poor. It is explained that the only gain margin or only phase margin is not a sufficient indicator of the robustness of the system.

For the stabilizer K_4, The gain margin and phase margin of the system are very large. But its open-loop amplitude-phase curve is very close to the critical point (-1, j0). It shows that the robustness of the system is poor. It shows that sometimes the combination of gain margin and phase margin may not be enough to indicate the real robustness of the system.

The problem is that the gain margin and phase margin can not correctly reflect the shortest distance from the critical point (-1, j0) of the open-loop amplitude-phase curve.

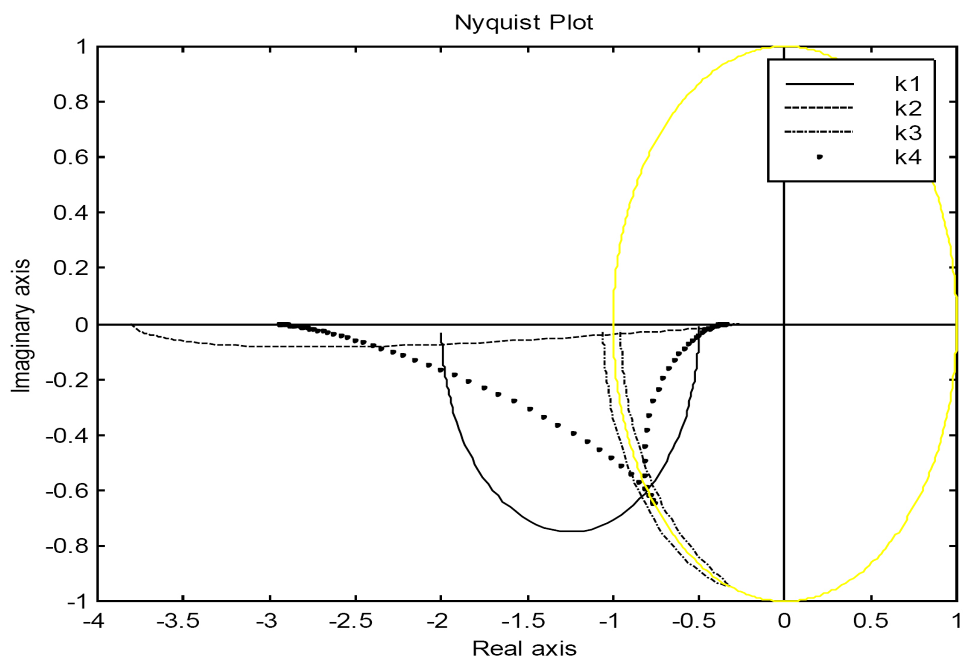

Figure2. *Nyquist diagrams of four systems*

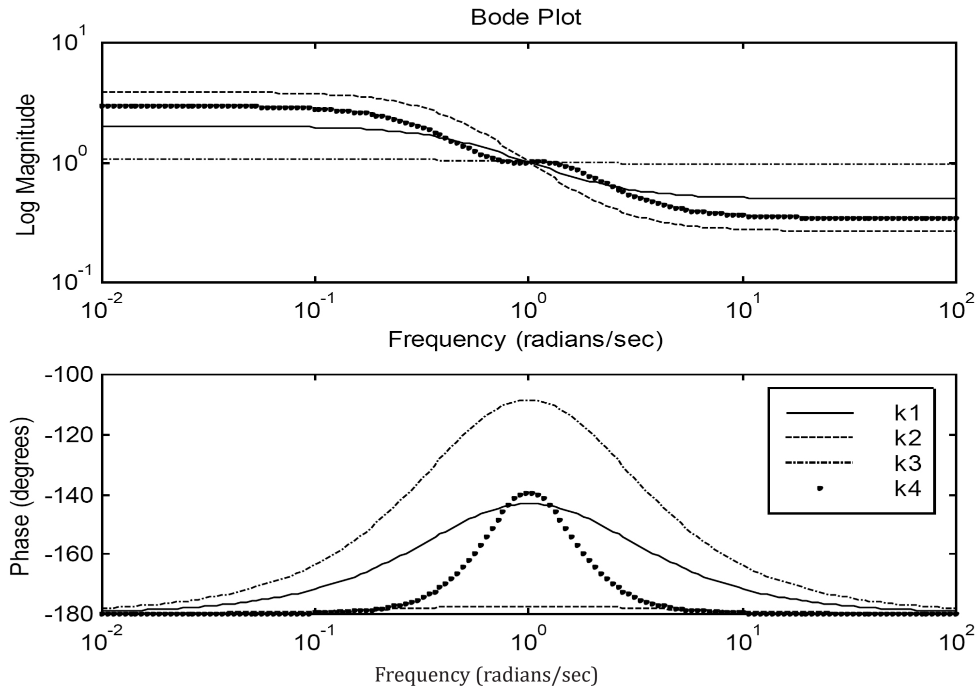

Figure3. *Bode diagrams of four systems*

3. For MIMO Systems

For MIMO systems, the robustness of the system can not be guaranteed even if its characteristic trajectory (equivalent to Nyquist diagram or open-loop amplitude-phase curve of SISO system) is far from the critical point (-1, j0). The following example illustrates this well.

Example 2: The transfer function matrix for the MIMO control system is:

$$\mathbf{G}(s) = \frac{1}{(s+1)(s+2)}\begin{bmatrix} -47s+2 & -56s \\ 42s & 50s+2 \end{bmatrix}$$

By introducing $\mathbf{V} = \begin{bmatrix} 7 & 8 \\ 6 & 7 \end{bmatrix}, \mathbf{V}^{-1} = \begin{bmatrix} 7 & -6 \\ -6 & 7 \end{bmatrix}$, make $\mathbf{G}(s)$ diagonally dominant

$$\mathbf{G}_1 = \mathbf{V}\mathbf{G}\mathbf{V}^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{2}{s+2} \end{bmatrix}$$

$\mathbf{G}_1$ is completely decoupled. Its characteristic function is:

$$g_1(s) = \frac{1}{s+1}$$

$$g_2(s) = \frac{2}{s+2}$$

The system's characteristic trajectory is the circle on the right half-complex plane (see Figure 4). It is impossible to touch the left half plane, let alone close to the (-1, j0) point. The control system corresponding to each diagonal element has infinite gain margin and 90 degree phase margin. From the point of view of SISO stability margin, the system should be very stable. However, if the first column of G (s) is multiplied by 1.07 (gain increase 7%) and the second column is multiplied by 0.93 (gain decrease 7%), the system becomes closed-loop unstable.

This shows that the Nyquist theorem of multivariable systems is not suitable for judging the robustness of multivariable systems. Analysis of the MIMO system using the SISO method may lead to incorrect conclusions.

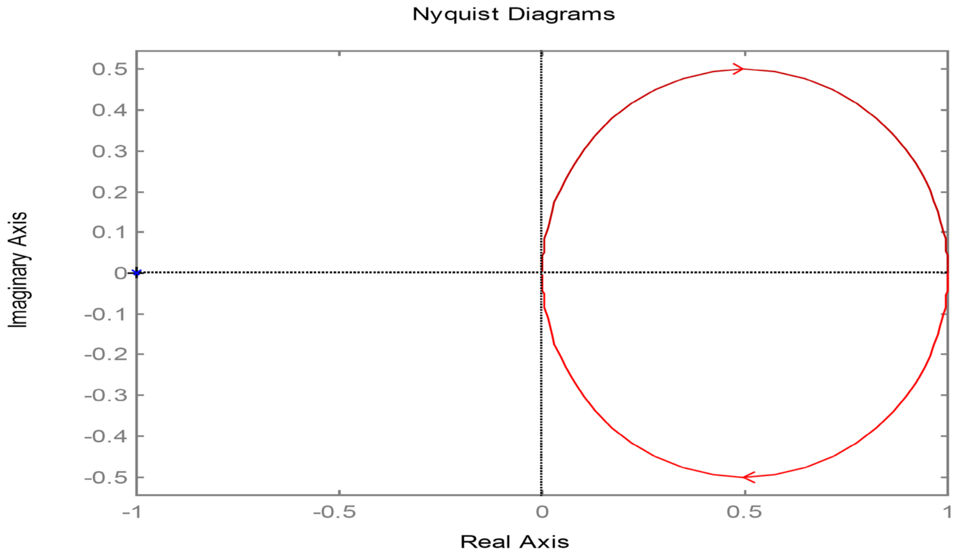

Figure4. *Nyquist diagram of example 2 system*

Conclusion

As can be seen from the above, the classical stability margin has the following lack:

1) For SISO systems, it does not reflect the minimum distance from the system Nyquist curve to the critical point (-1, j0). Or it can not reflect the influence of uncertainties that gain and phase change simultaneously on the system.

2) For MIMO systems, Even if each loop has a good stability margin, when both loops have uncertainties at the same time, the system can easily become unstable. It can not reflect the influence of multi-loop uncertainty on MIMO system.

In order to overcome these lacks, new concepts and indicators are needed to characterize the characteristics of the system.

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