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Every Even Integer Greater than 500000 can be Expressed as a Sum of Two Odd Primes

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Abstract: Every even integer greater than four can be expressed as a sum of two odd primes, and exists the formula as follows:

Gp(N) ≥ INT{ Kpc×Ctwin×N/(Ln N)^2 }-1 ≥INT{ 0.66016×N/(Ln N)^2 }^1 ≥1915 >>1

where the Gp(N) be the number of primes P with N-P primes, or, equivalently, the Gp(N) be the number of ways of writing N as a sum of two primes, the N be the even integer greater than 500000.

Keywords: Even integer, Goldbach prime, Goldbach's Conjecture.

One: The Proof Method of Goldbach's Conjecture

The Goldbach's Conjecture is one of the oldest unsolved problems in Number Theory. In its modern form, it states that every even integer greater than two can be expressed as a sum of two primes.

Let N be an even integer greater than 2, and let N = (N-Gp)+Gp, with N-Gp and Gp prime numbers, the Gp{Gp≤N/2} be a Goldbach Prime of even integer N. Let Gp(N) be the number of Goldbach Primes of even integer N. The number of ways of writing N as a sum of two prime numbers, when the order of the two primes is important, is thus GP(N) = 2Gp(N) when N/2 is not a prime and is GP(N) = 2Gp(N)-1 when N/2 is a prime. The Goldbach's Conjecture states that Gp(N) > 0, or, equivalently, that GP(N) > 0, for every even integer N greater than two.

We known that the Goldbach's Conjecture is true for every even integer N no greater than 30000, therefore, we only need to prove that the Goldbach's Conjecture is true for every even integer N greater than 30000, that is: Gp(N|N >30000) ≥1.

Two: The Sieve Method About The Goldbach Primes

Let N be an even integer greater than 30000, then the even integer N can be expressed to the form as follows:

$$N = (N - Gn) + Gn, \quad Gn \le N / 2 \qquad (1)$$

where Gn be the positive integer no greater than N/2.

Sieve Method

Let N-Gn and Gn are two positive integers, if N-Gn and Gn any one can be divisible by the prime P, then sieves the positive integer Gn; if both the N-Gp and Gp can not be divisible by the all primes no greater than √N, then both the N-Gp and Gp are primes at the same time, the prime Gp be called the Goldbach Prime of even integer N.

Theorem One

Let Pc be an odd prime factor of even integer N and no greater than √N, then the ratio of the number of integers Gp that both the N-Gp and Gp can not be divisible by the prime Pc to the total of integers Gn no greater than

N/2 is follows:

R(N,Pc) = INT{ N/2 - N/2/Pc } / (N/2) / { INT(N/2) - INT(N/2/Pc) } / (N/2)

Proof

Because Pc is an odd prime factor of even integer N, therefore, both the N-Gn and Gn can or can not be divisible by prime Pc at the same time, then the number of integers Gn that the N-Gn and Gn any one can be divisible by the prime Pc is INT{(N/2)/Pc}, the number of integers Gn that both the N-Gn and Gn can not be divisible by the prime Pc is { INT(N/2) - INT(N/2/Pc) } = INT{ N/2-N/2/Pc }, the ratio of the number of integers Gn that both the N-Gn and Gn can not be divisible by the prime Pc to the total of integers Gn no greater than N/2 is follows:

R (N,Pc) = { INT(N/2) - INT(N/2/Pc) } / (N/2) = INT{ N/2 - N/2/Pc } / (N/2) (2)

Theorem Two

Let Pn be an odd prime no factor of even integer N and no greater than √N, then the ratio of the number of integers Gn that both the N-Gn and Gn can not be divisible by the prime Pn to the total of integers Gn no greater than N/2 is follows:

R(N,Pn) = INT{ N/2 - N/Pn } / (N/2) = { INT(N/2) - INT(N/Pn) } / (N/2)

Proof

Because the Pn is an odd prime no factor of even integer N, therefore, both the N-Gn and Gn can not be divisible by the prime Pn at the same time, that is the N-Gn and Gn only one can be divisible or both the N-Gn and Gn can not be divisible by the prime Pn, then the number of integers Gn that the N-Gn and Gn any one can be divisible by the prime Pn is INT{N/Pn}, the number of integers Gn that both the N-Gn and Gn can not be divisible by the prime Pn is { INT(N/2) - INT(N/Pn) } = INT{ N/2 - N/Pn }, the ratio of the number of integers Gn that both the N-Gn and Gn can not be divisible by the prime Pn to the total of integers Gn no greater than N/2 is follows:

R(N,Pn) = { INT(N/2) - INT(N/Pn) } / (N/2) = INT{ N/2 - N/Pn } / (N/2) (3)

Theorem Three

The integer 2 is an even prime factor of even integer N, the ratio of the number of integers Gn that both the N-Gn and Gn can not be divisible by the even prime 2 to the total of integers Gn no greater than N/2 is follows:

R(N,2) = INT{ N/2-N/2/2 } / (N/2) = { INT(N/2) - INT(N/2/2) } / (N/2)

Proof

Because the 2 is an even prime factor of even integer N, therefore, both the N-Gn and Gn can be divisible or can not be divisible by the even prime 2 at the same time, then the number of integers Gn that the N-Gn and Gn any one can be divisible by the even prime 2 is INT{ N/2/2 }, the number of integers Gn that both the N-Gn and Gn can not be divisible by the even prime 2 is { INT(N/2) - INT(N/2/2) } =INT{ N/2 - N/2/2 }, the ratio of the number of integers Gn that both the N-Gn and Gn can not be divisible by the even prime 2 to the total of integers Gn no greater than N/2 is follows:

R(N,2) = { INT(N/2) - INT(N/2/2) } / (N/2) = INT{ N/2 - N/2/2 } / (N/2) (4)

Three: The Number of Goldbach Primes of Even Integer

Let Gp(N) be the number of Goldbach primes of even integer N, let Gp(N,Pn) be the number of Goldbach primes no greater than √N, then exists the formulas as follows:

Gp(N) = INT{(N/2)×R(N,2)×∏R (N,Pci)×∏R (N,Pni)} + Gp(N,Pni) - 1(if N-1 prime)

$= INT\{(N/2)\times(1-1/2)\times\prod(1-1/Pci)\times\prod(1-2/Pni)\}+Gp(N,Pni)-1(\text{if } N-1 \text{ prime})$ (5)

Where Pci and Pni are odd primes no greater than $\sqrt{N}$.

Let Pi(N) be the number of primes less than an integer N, then, be the formula as follows:

$Pi(N) \equiv INT\{ N\times(1-1/P1)\times(1-1/P2)\times\ldots\times(1-1/Pm)+m-1 \} \equiv P(N)+Pi(\sqrt{N}) - 1$

$Pi(N) \approx Psha(N)\equiv Li(N)- 1/2 \times Li(N^{0.5})$

$P(N|N\geq10^9) \geq 2 /(1+\sqrt{(1-4 / Ln(N))})\times N / Ln(N)\geq N /(Ln(N)-1)$

$P(N|N\geq10^4) \equiv INT\{ N\times(1-1/2)\times\prod(1)1/Pi)\} \geq N / Ln(N)$ (6)

Four: The Proof of Goldbach's Conjecture

Theorem Four

Every even integer greater than 500000 can be expressed as a sum of two odd primes.

Proof

According to the formula (5), we can obtain the formula as follows:

$Gp(N)+ 1 \geq INT\{(N/2)\times(1-1/2)\times \prod(1-1/Pci)\times \prod(1-2/Pni)\}$

$= INT\{(N/2)\times(1-1/2)\times \prod((Pci-1)/(Pci-2))\times\prod(1-2/Pci)\times\prod(1-2/Pni)\}$

$= INT\{(N/2)\times(1-1/2)\times \prod((Pci-1)/(Pci-2))\times \prod(1-2/Pi)\}$

$= INT\{(N/2)\times(1-1/2)\times Kpc \times \prod(1-2/Pi)/ \prod(1-1/Pi)^2 \times \prod(1-1/Pi)^2 \}$

$= INT\{(N/2)\times(1-1/2)\times Kpc \times \prod(1-1/(Pi-1)^2)\times \prod(1-1/Pi)^2 \}$

$\geq INT\{(N/2)\times(1-1/2)\times Kpc \times Ctwin \times \prod(1-1/Pi)^2 \}$ (7)

Apply the formula (6), we can obtain the formula as follows:

$Gp(N|N\geq500000)\geq INT\{(N/2)\times(1-1/2)\times Kpc \times Ctwin \times \prod(1-1/Pi)^2 \} - 1$

$\geq INT\{ Kpc \times Ctwin \times N / Ln(N)^2 \} - 1 \geq INT\{ 0.6601618159 \times N / Ln(N)^2 \} - 1$

$\geq INT\{ 0.6601618159\times(500000)/ Ln(500000)^2 \}-1 = INT\{ 1916.89 \}-1 = 1915 >> 1$ (8)

From above formula (8) we can obtain that:

Every even integer greater than 500000 can be expressed as a sum of two odd primes.

Conclusion

The Goldbach's Conjecture is a Complete Correct Theorem. Proof end.

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