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Interplay of SNR with Diversity for Minimum Mean Squared Error Receiver

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Abstract: Poor performance in wireless channel arises due to the Deep Fade and probability of deep fade in the system is just the reciprocal of the SNR (1/SNR). Solution to this problem lies in use of diversity i.e. using more links. That could be achieved by the use of multiple transmit and receive antennas. At the receiver multiple received signals are available as a linear combination of individual signals. These are used at the input of detection in the form of Beam forming vector. Beam forming vector is a vector combination of the receive signals. Noise component at the receiver is a random quantity that depends on the Norm of the vector of the noise at each receive antenna. To maximize the SNR, we may choose appropriate Beam forming vector. The combiner that provides maximum SNR under such conditions is referred as Maximal Ratio Combiner. This is a scaled version of fading channel vector (spatial matched filter). Receiver diversity is successfully employed in WCDMA. HSDPA, LTE and WiMAX technologies. BER performance of the multiple antenna system follows the Chi square distribution. As receive antennas increase, the probability of deep fade and hence BER also decreases at a much faster pace. MIMO systems evolve in finding the minimum error vector amongst all possible transmit vectors. There are attempts to provide a solution that minimizes the least square error as implemented in Zero Forcing Receivers (ZFR). It uses pseudo inverse to arrive at the ZFR diversity orders in terms of number of receive and transmit antenna. On the other hand we analyze the Minimum Mean Squared Error (MMSE) receiver which calculates the mean square of the error following Bayesian approach which is different from earlier case of ZFR where we considered the deterministic error. In this paper we look in to the conditions as to how these two receivers operate and conditions under which these converge. Also what could be reason for the noise enhancement in ZFR and how MMSE improves on this drawback, has been discussed. The requirement of SNR for various channels and its shortfall has been analytically presented. Impact of diversity over the SNR requirement has been modeled and same was simulated to verify the SNR shortfall in case of various MIMO channels.

Keywords: Minimum Mean Squared Error (MMSE), Zero Forcing Receiver (ZRF), SNR, Single Value Decomposition, MIMO, Single Value Decomposition (SVD)

INTRODUCTION

Very high SNR is needed to achieve a high BER in case of any wireless channel as compared with the wired channel. Wireless channels thus have a challenge to overcome channel losses and cater to the mobility of transmitter and the receiver. Hence such a disparity had to be overcome to ensure survival of wireless technology. Fading is a common problem and condition of deep fade is always a concern. To overcome it, diversity was employed by setting multiple transmit and receive antennas. Deep fade can be addressed using special receivers. Zero Forcing Receivers (ZFR) and Minimum Mean Squared Error (MMSE) receivers are two important concepts that overcome the error due to deep fade either in deterministic manner or by the use of mean square of the error. A survey on enhancing the LTE limits was undertaken in [1]. Employment of TD-SCDMA protocol was

explored in [2]. Kun Wang et el [3] has discussed the problem of compact arrays whose inter-element distance is smaller than half of the operative wavelength. These cases are very peculiar in a way that decoupling of channels becomes a challenge for obtaining higher performance. In simulated results [4] it is shown that Zero Forcing Equalizer removes Inter Symbol Interference and is best suited for noiseless conditions of a channel. In case of noisy channels, such receivers tend to enhance the noise for frequencies where channel response is low. MIMO aims to increase transfer rates using Spatial Multiplexing [5].

ZERO FORCING RECEIVER

MIMO system is represented as one having multiple transmit (say 't') and receive (say 'r') antennas. MIMO linear receiver can be defined as $y = Hx + n$ where $x_1, x_2, \dots x_t$ are the transmitted symbols, n is the channel noise and received symbols can be represented as $y_1, y_2, \dots y_r$ etc. The symbols represent the fading coefficient between transmit antenna and receive antenna. If we take the inverse on two sides then: $. H^{-1} y = x + H^{-1} n$. Inverse will exist only in case of square matrix (r= t) number of transmit antennas equals number of receive antennas and matrix should have full rank). Among all possible vectors we choose '' in such a way that error vector i.e.

$$\text{Error} = \| y - Hx \|^2$$

is minimized. So we can get approximate solution for the least possible squared error. Norm square of the vector can be mathematically written as vector transpose multiplied by the vector.

$$\| y - Hx \|^2 = (y - Hx)^T (y - Hx)$$

$$= \bar{y}^T \bar{y} - \bar{x}^T H^T \bar{y} - \bar{y}^T H \bar{x} + \bar{x}^T H^T H \bar{x}$$

$$= 2\bar{x}^T H^T \bar{y} + 2H^T H \bar{x} \qquad (1)$$

To apply the concept of maxima and minima, we differentiate the $\| y - Hx \|^2$ with respect to to get:

$$\frac{d \| y - Hx \|^2}{dx} = -2H^T \bar{y} + 2H^T H \bar{x}$$

For a minima, we arrive at:

$$H^T \bar{y} = H^T H \bar{x}$$

Estimate of x in case of number of transmit antenna is more that the receive antenna which is represented as Zero Forcing Receiver.

$$\bar{x} = (H^T H)^{-1} H^T \bar{y} \qquad (2)$$

For complex notations we may interpolate the equation by replacing the Transpose with Hermitian operator to determine the pseudo inverse of the matrix as under:

$$\bar{x} = (H^H H)^{-1} H^H \bar{y} \qquad (3)$$

Receivers based on the above the equation (3) is called a Zero Forcing Receiver (ZFR). ZFR has a problem that it tends to amplify the noise in case of low values of 'h' and hence Minimum Mean Squared Error receiver is preferred.

MMSE RECEIVER

MMSE receivers can be considered as an estimator which accepts random received symbols and estimates about the possible value of transmitted symbol. In this process of estimating the transmitted symbol, mean squared error is considered and effort is made to minimize it. To make computations more logical and trustworthy, pseudo-inverse of channel H is computed. Same problem could be modeled as an estimation of a scalar transmitted symbol 'x' when 'r' vector inputs of received are available. Diversity remains at the core of measurement. Receiver should have the capability to minimize mean squared error $E \{\|\mathbf{C}^T \bar{\mathbf{y}} - \mathbf{x}\|^2\}$. This can be re-written as:

$$\text{Expected error (squared)} = \{\bar{c}^T R_{yy}\, \bar{c} - R_{xy}\, \bar{c} - \bar{c}^T R_{yx} + R_{xx}\}$$

where $R_{xy} = E\,(x\,\bar{y}^T) = E\,(\bar{y}\,x^T) = R_{yx}$ are cross covariance.

And hence a combiner can be suitably designed to estimate squared error as:-

$$E = \bar{c}^T R_{yy}\, \bar{c} - 2.\, \bar{c}^T R_{yx +}\, R_{xx}$$

Applying the principle of minima for above expression, we may arrive at $R_{yy}\, \bar{c} = R_{yx.}$ This shows the value of $\bar{c}$ for minimum error as:

$$\bar{c} = Ryy^{-1} . R_{yx}$$

Hence estimated value of transmitted symbol the transmitted symbol 'x':

$$\hat{x} = \bar{c}^T. \bar{y}$$

in case of MIMO $\bar{c}$ is a matrix. This can be generalized for the complex space. In case and being vectors similar relation holds.

Covariance of the transmit symbols can be shown as $R_{xx} = \begin{pmatrix} |x_1|^2 & x_1 x_2^* \cdots & x_1 x_t^* \\ x_2 x_1^* \vdots & \ddots & \vdots \\ x_t x_1^* & \cdots & |x_t|^2 \end{pmatrix}$

All cross correlation terms are zero and hence the above matrix will reduce to

$$R_{xx} = \begin{pmatrix} P_d & 0 \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & P_d \end{pmatrix}$$

Here P_d is the transmitted symbol power.

$$R_{xx} = P_d I_t$$

here I_t is the identity matrix. In similar way we can compute covariance of the received symbols as under:

$$R_{yy} = E\,(\bar{y}\,\bar{y}^H)$$

$$= E\,\{(H\bar{x} + \bar{n})(H\bar{x} + \bar{n})^H\}$$

$$= E\,\{(H\bar{x}\,\bar{x}^H H^H + \bar{n}\,\bar{x}^H H^H + H\bar{x}\bar{n}^H + \bar{n}\bar{n}^H)\}$$

$$R_{yy} = H R_{xx} H^H + \sigma^2 I$$

$$= P_d H H^H + \sigma_n^2 I$$

Now in order to calculate

$$R_{yy} = E\,(\bar{y}\,\bar{x}^H)$$

$$= E\,\{(H\bar{x} + \bar{n})\,\bar{x}^H)\}$$

$$= P_d H$$

Hence Linear Minimum Mean Squared Error estimator for the transmitted symbol for MIMO system

$$\hat{x} = \bar{c}^H \bar{y} = P_d H^H\,(HH^H + \sigma_n^2 I)^{-1}\,\bar{y} \qquad (4)$$

AVERAGE DELAY SPREAD

Multiple transmitters and receivers further complicate the accounting of delays observed in various scattered and direct paths. Various rays Delay could be attributed to the time difference between the scattered paths and direct path. On an average for a cell of 2 Km radius it could be assumed that difference in the path length could be of the order of ≈ Km. Considering a mobile at the edge of a cell (i.e. at a distance of approximately 2 Km from the Base station. Hence direct path can be assumed to be 2 Km and indirect paths could be longer say 3 Km, 4 Km or on similar lines.

Hence delays τ_0, τ_1, τ_2 ...can be computed as τ_0 = 2Km/c, $_1$=3Km/c, $_2$=4Km/c...etc where 'c' is speed of light. We may assume that delay spread to be of the order of 1Km/3 x 10^8 ≈3.33 μS. We see the delay spread in 3G/4G systems is of the order of 1-3 μS.
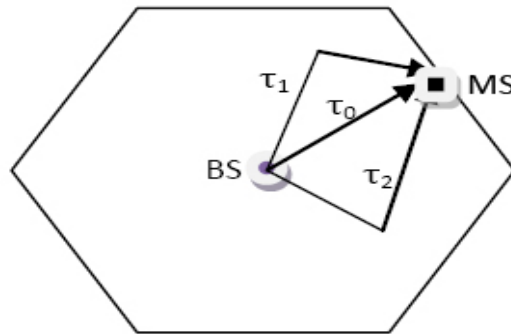

Fig1. *Delay spread of Mobile station in a GSM Cell*

SINGLE VALUE DECOMPOSITION

The process of Single Value Decomposition (SVD) is used to perform transmission and reception using MIMO communication system [6]. Spatial multiplexing has been discussed as transmission is allowed through multiple spatial channels [7]. This is also referred to as diversity and multiple transmitter and receiver antennas are employed to achieve capacity. SVD is useful in removing co-channel interference by distributing MIMO channels into individual SISO channels which may not be correlated [8]. SVD fading channel could be represented as:

$$H = u \sum V^H.$$

Where received symbol is y = H.x + n and H is MIMO channel matrix, 'x' is MIMO transmit vector and $\bar{n}$=average noise. We can replace H using SVD as:

$$y = u \sum V H . \bar{x} + \bar{n}$$

if we attempt pre-coding at the transmitter and consider multiple beam forming at the receiver

$$\boldsymbol{u^H \bar{y}} = \tilde{y} = u^H (u \sum V^H . \bar{x} + \bar{n}) \qquad (5)$$

It reduces to:

$$\tilde{y} = \sum V^H . \bar{x} + u^H \bar{n},$$

if we pre-code the symbols at transmitter in such a way that $\bar{x} = V\tilde{x}$ then expression (5) reduces to

$$\tilde{y} = \sum \tilde{x} + \tilde{n})$$

hence just by replacing the H with its SVD $(u \sum V^H)$ and pre-coding at the transmitter $(\bar{x} = V\tilde{x})$, system model reduces to

$$\tilde{y} = \begin{pmatrix} \bar{y}_1 \\ \bar{y}_2 \\ . \\ . \\ . \\ . \\ \bar{y}_t \end{pmatrix} = \begin{pmatrix} \sigma_1 & 0 \cdots & 0 \\ \vdots & \sigma_2 \ddots & \vdots \\ 0 & \cdots & \sigma_t \end{pmatrix} \text{x} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ . \\ . \\ . \\ . \\ \tilde{x}_t \end{pmatrix} + \begin{pmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ . \\ . \\ . \\ . \\ \tilde{n}_t \end{pmatrix}$$

In case SVD was not applied, all the transmit symbols interfered with every receive antenna but due to SVD we get

$$\tilde{y}_i = \sigma_i \tilde{x}_i + \tilde{n}_i \tag{6}$$

hannel is σ_i. Hence if we know $\tilde{y}_i$, $\tilde{x}_i$ can be estimated. Assume we transmit 't' symbols in parallel then we may extend the argument to MIMO. Equation (6) represents decoupling of the channel. This may be referred as parallelization. Here received symbol is an aggregation of 't' parallel channels where gain in the ith
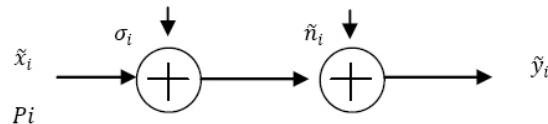
SVD thus helps in decoupling the interference based system in to independent channels.

MIMO CHANNEL SNR REQUIREMENT

Spectrum efficiency and Energy efficiency relationship was discussed in [9], It was reported that Energy Efficiency has exponential dependence over the linear variations in System Efficiency. Hence MIMO may not be energy efficient as linear increase in Spectrum Efficiency causes an exponential decrease of Energy Efficiency. Capacity of the OFDM system was critically anslysed in [10] and [11]. To understand what rate the channel of transmission can support we may analyze the SNR requirements of ith channel.

$$\text{SNR for } i^{th} \text{ channel} = \text{SNR}_i = \frac{P_i \sigma_i^2}{\sigma_n^2}$$

SNR limits the channel capacity to $\log_2 (1 + \text{SNR})$. Hence maximum throughput for i[th] channel will be limited to

$$C_i^{th} = \log_2 \left(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}\right) \tag{7}$$

For MIMO channel the capacities of all 't' channels are added:

$$C = \sum_1^t \log_2 \left(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}\right) \tag{8}$$

To ensure maximum throughput, we need to maximize C, when maximum power at transmitter (P) is divided among 'i' channels. The aim is to allocate transmitter power in such a way that throughput is maximum hence $P_i \leq P$ is the limiting factor. Diversity is the key factor to guarantee outperformance of combination of OFDM and MIMO over traditional OFDM system that may use one antenna for transmit and receive [3].

$$\frac{dC}{dt} = 0 \Rightarrow \frac{\sigma_i^2 / \sigma_n^2}{1 + Pi\left(\sigma_i^2 / \sigma_n^2\right)} + \lambda(-1) = 0$$

Here λ is Lagrange multiplier, we get

$$P_i = (\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2}) \qquad (9)$$
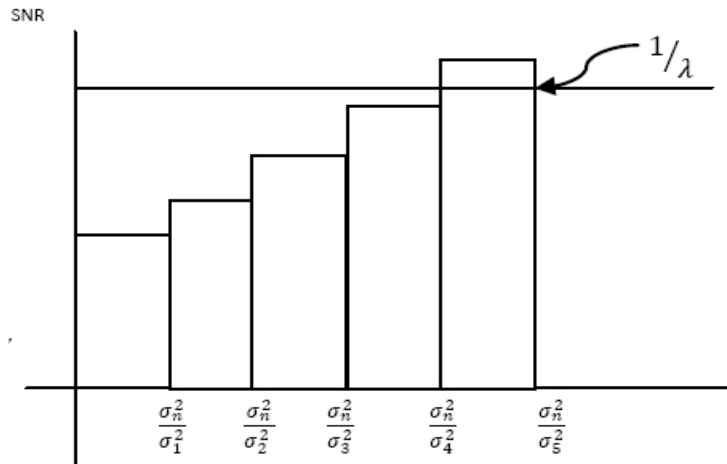
Fig2. *shortfall of SNR in step value as compared with 1λ*

power is to be added to a particular MIMO channel.

$$\sum_i P_i = \sum_i (\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2}) = P$$

$$P_i = (\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2}) \quad \text{for i= 1, 2, 3,...}$$

$$P_1 = (\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_1^2})$$

$$P_2 = (\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_2^2})$$

And so on. For all the channels that have the power allocation level < $1/\lambda$, here will be a need to add power to equalize the shortfall. This is popularly known as Water filling algorithm. If all channels have non zero positive power then the requirement is met and it will be optimal for that set of MIMO channels having non zero singular values.
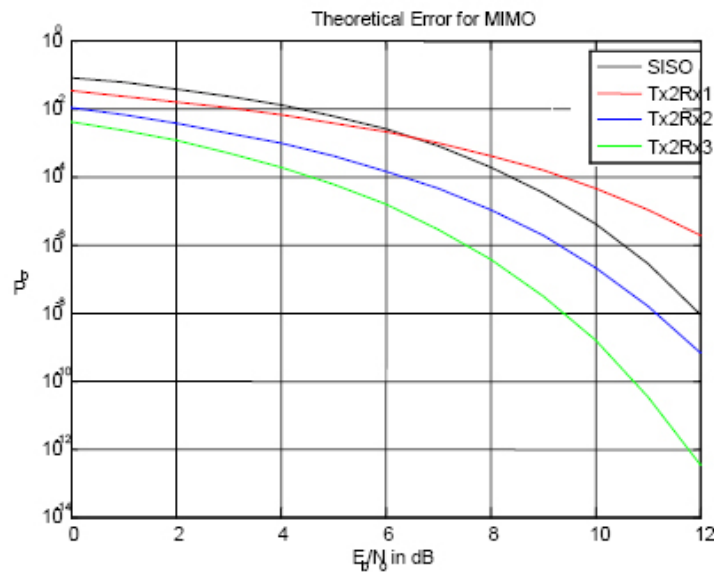
SIMULATION

Theoretical plot of AWGN and Rayleigh channel (Fig.-3) indicates the Rayleigh channel performance as compared with usual AWGN channel. Rayleigh channel experiences sharp fading as compared with the AWG channel. At a BER of 10^{-3} Eb/No sharply declines approximately to 11 dB whereas in case of AWGN channel fading is moderate and Eb/No value remains a modest 30 dB. SISO case follows the Rayleigh fading channel conditions. The point that emerges is the need to address the steep fall in SNR for MIMO channels by the use of diversity.
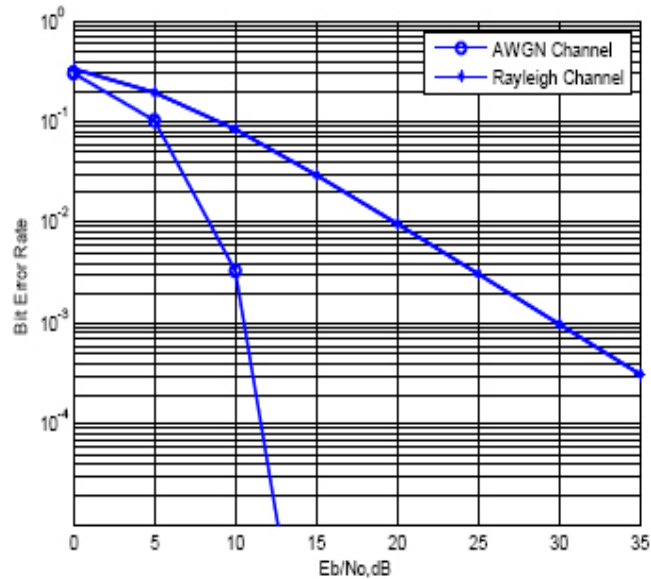
Fig3. *SNR variation for AWGN and Rayleigh Channel (SISO)*

SNR performance related to various diversity combinations have been modeled in the Matlab simulation. Large numbers of frames (=100000) were analysed among 256 carriers. Data symbol of 192 bit size was considered. Various transmit and receive antenna combinations have been simulated to arrive at the situation where bit energy has been compared with bit error. Theoretical results are presented in Fig. 4. Diversity combinations were simulated and results are shown in Fig.5. Clear advantage of diversity achieved using combination of Two Transmit- One Receive antennas (Tx2Rx1), Two Transmit- Two Receive antennas (Tx2Rx2) and Two Transmit-Three Receive antennas (Tx2Rx3) have been indicated. There is a clear outperformance of diversity over SISO (One Transmit- One Receive antenna: Tx1Rx1).

Fig4. *Comparison of Bit errors vs energy requirement for different cases of MIMO*

Fig. 6 has represented the SNR advantage of the combination in bar chart. The simulaton has produced the chart similar to as expected in the theoretical domain (Fig. 2) indicating the shortfall in the SNR with change in diversity. The maximum value that is desirable equals $1/\lambda$.
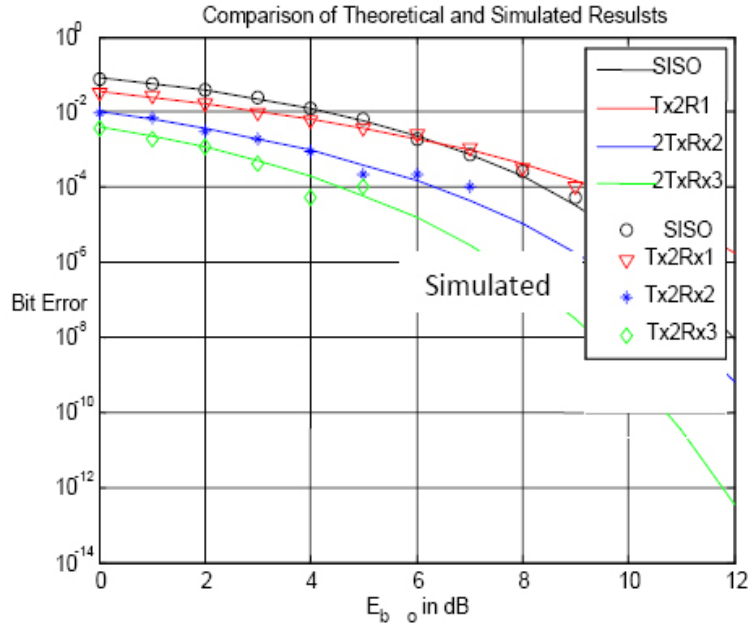

Fig5. *Comparison of SNR performance for MIMO theoretical and simulated results*
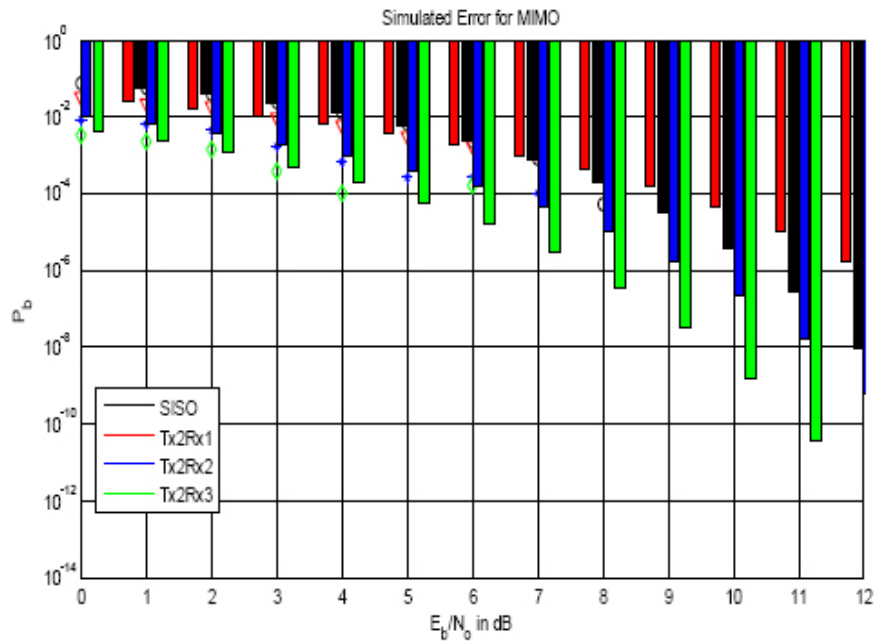

Fig6. *Outperformance of increasing Diversity among Transmit and Receive antennas*

Conclusion

This paper has brought out a clear influence of diversity with SNR of received symbols in case of MIMO. The shortfall of SNR as the diversity changes also confirms theoretical relation (9).

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